

Precision Tests of Flavor and CP Violation in B Decays ^{*}

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Isospin and flavor SU(3) set stringent bounds on penguin pollution in $B^0(t) \rightarrow \rho^+ \rho^-$, providing a theoretically precise determination of $\alpha \equiv \phi_2$, $\alpha = (91 \pm 7_{\text{exp}} \pm 3_{\text{th}})^\circ$. Isospin breaking in a sum rule for $B \rightarrow K\pi$ rates is shown to be suppressed. A similar sum rule holds for CP asymmetries in $B \rightarrow K\pi$. Violation of these sum rules would be evidence for an anomalous $\Delta I = 1$ piece in \mathcal{H}_{eff} .

1. INTRODUCTION

Two major purposes of high statistics experiments studying B and B_s decays in e^+e^- and hadron colliders are: (1) Achieving great precision in Cabibbo-Kobayashi-Maskawa parameters [1], in particular determining the CP violating phase, the source of CP violation in the Standard Model. (2) Identifying potential inconsistencies by over-constraining these parameters. For instance, the phase $\beta \equiv \phi_1 \equiv \arg(-V_{cb}^* V_{cd}/V_{tb}^* V_{td}) = (21.7^{+1.3}_{-1.2})^\circ$, measured very precisely in time-dependent CP asymmetries of B^0 decays via $b \rightarrow c\bar{c}s$ [2,3], may be tested also in $b \rightarrow sq\bar{q}$ ($q = u, d, s$) penguin-dominated decays [4,5] which are susceptible to effects of physics beyond the Standard Model [6]. Alternatively, a violation of precise symmetry relations among certain observables in $b \rightarrow sq\bar{q}$ transitions could provide unambiguous evidence for new physics.

In this talk I wish to discuss a few examples for achieving these two goals, where important progress has been made recently, both theoretically and experimentally. In Section 2 I focus on the currently most precise determination of $\alpha \equiv \phi_2 \equiv \arg(-V_{tb}^* V_{td}/V_{ub}^* V_{ud})$, based primarily on $B \rightarrow \rho^+ \rho^-$. While the traditional method is based on isospin symmetry [7], I will argue for an advantage of using flavor SU(3) to set bounds on

the penguin amplitude contributing to this process [8]. In Section 3 I study two precision isospin sum rules, for $B \rightarrow K\pi$ decay rates [9,10] and for the CP asymmetries in these decays [11,12]. Isospin breaking corrections to the sum rule for rates will be shown to be suppressed by the small ratio of tree and penguin amplitudes contributing in these decays [13]. The sum rule involving four CP asymmetries [12] (or three asymmetries [11]) replaces a much less accurate relation, $A_{\text{CP}}(K^+ \pi^0) \sim A_{\text{CP}}(K^+ \pi^-)$ [9], which is sometimes being claimed to hold in the Standard Model but seems to be violated experimentally.

2. PRECISION DETERMINATION OF α

2.1. $B \rightarrow \pi\pi$

The amplitude for $B^0 \rightarrow \pi^+ \pi^-$ contains two terms [4,5], conventionally denoted “tree” (T) and “penguin” (P) amplitudes, involving a weak phase γ and a strong phase δ :

$$A(B^0 \rightarrow \pi^+ \pi^-) = T e^{i\gamma} + P e^{i\delta}. \quad (1)$$

We use the c -convention [14], in which the top-quark in the $\bar{b} \rightarrow \bar{d}$ loop has been integrated out and the unitarity relation $V_{tb}^* V_{td} = -V_{cb}^* V_{cd} - V_{ub}^* V_{ud}$ has been used. A rather large value, $P/T \sim 0.5$, is implied [15] by comparing within flavor SU(3) the measured rate for this process with that measured for $B \rightarrow K\pi$.

Time-dependent decay rates, for an initial B^0 or a \bar{B}^0 , are given by [5]

$$\Gamma(B^0(t)/\bar{B}^0(t) \rightarrow \pi^+ \pi^-) \propto e^{-\Gamma t} \Gamma_{\pi^+ \pi^-} \times$$

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$$[1 \pm C_{+-} \cos \Delta mt \mp S_{+-} \sin \Delta mt], \quad (2)$$

$$S_{+-} = \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{+-} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad (3)$$

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+\pi^-)}{A(B^0 \rightarrow \pi^+\pi^-)}. \quad (4)$$

The three measurables, $\Gamma_{\pi^+\pi^-}$, S_{+-} and C_{+-} are insufficient for determining T, P, δ and γ or α .

The isospin method [7] is based on obtaining additional information from two isospin triangle relations for B and \bar{B} ,

$$A(\pi^+\pi^-)/\sqrt{2} + A(\pi^0\pi^0) - A(\pi^+\pi^0) = 0. \quad (5)$$

Defining $\sin 2\alpha_{\text{eff}} \equiv S_{+-}/(1 - C_{+-}^2)^{1/2}$, the difference $\theta \equiv \alpha_{\text{eff}} - \alpha$ is determined up to a sign ambiguity by constructing the two isospin triangles with a common base, $A(\pi^+\pi^0) = A(\pi^-\pi^0)$. A small electroweak penguin amplitude creates a calculable angle between the two basis, implying a calculable correction in the extracted value of α [16], $\Delta\alpha_{\text{EWP}} = -1.5^\circ$.

In the absence of separate branching ratio measurements for $B^0 \rightarrow \pi^0\pi^0$ and $\bar{B}^0 \rightarrow \pi^0\pi^0$, the strongest upper bound on $|\theta|$ in terms of CP-averaged rates and a direct CP asymmetry in $B^0 \rightarrow \pi^+\pi^-$ is given by [17]

$$\cos 2\theta \geq \frac{(\frac{1}{2}\Gamma_{+-} + \Gamma_{+0} - \Gamma_{00})^2 - \Gamma_{+-}\Gamma_{+0}}{\Gamma_{+-}\Gamma_{+0}\sqrt{1 - C_{+-}^2}}. \quad (6)$$

Somewhat weaker bounds contained in this bound were derived in Ref. [18]. A complete isospin analysis requires measuring also $C_{00} \equiv -A_{CP}(\pi^0\pi^0)$, the direct asymmetry in $B^0 \rightarrow \pi^0\pi^0$.

Current asymmetry measurements [3], $S_{+-} = 0.50 \pm 0.12$, $C_{+-} = -0.37 \pm 0.10$, and corresponding branching ratio measurements, imply $\alpha_{\text{eff}} = (106 \pm 5)^\circ$, $|\theta| < 36^\circ$. Two properties, $P/T \leq 1$, $|\delta| \leq \pi/2$, confirmed experimentally in a global parameter fit, have been shown to resolve a sign ambiguity in θ [19], thereby implying $\alpha < \alpha_{\text{eff}}$ and consequently

$$\alpha = (88 \pm 18)^\circ. \quad (7)$$

2.2. Isospin in $B \rightarrow \rho\rho$

Angular analyses of the pions in ρ decays have shown that $B^0 \rightarrow \rho^+\rho^-$ is dominated by longitudinal polarization [3], $f_L = 0.97_{-0.03}^{+0.02}$. This simplifies the study of CP asymmetries in these decays (an example of $B \rightarrow VV$) to becoming similar to $B^0 \rightarrow \pi^+\pi^-$ (an example of $B \rightarrow PP$). The advantage of $B \rightarrow \rho\rho$ over $B \rightarrow \pi\pi$ is the smallness of $\mathcal{B}(\rho^0\rho^0)$ ($\mathcal{B}(\rho^0\rho^0) < 1.1 \times 10^{-6}$) relative to $\mathcal{B}(\rho^+\rho^-)$ and $\mathcal{B}(\rho^+\rho^0)$ [both in the range $(20 - 30) \times 10^{-6}$] in comparison with the corresponding relative branching ratios in $B \rightarrow \pi\pi$. The smaller P/T ratio in $B \rightarrow \rho^+\rho^-$, $P/T < 0.3$ [20], leads to a stronger upper bound $|\alpha - \alpha_{\text{eff}}| < 11^\circ$ in $B \rightarrow \rho\rho$. The asymmetry measurements [3], $S_L = -0.21 \pm 0.22$, $C_L = -0.03 \pm 0.17$, lead to $\alpha_{\text{eff}} = (96_{-6}^{+7})^\circ$. Thus, one finds $\alpha = (96 \pm 13)^\circ$ by adding errors in quadrature. This error is dominated by the current uncertainty in $\alpha - \alpha_{\text{eff}}$ including an ambiguity in its sign.

The error in α may be reduced by making one very reasonable and mild assumption about the strong phase, $|\delta| \leq \pi/2$. This is motivated by QCD factorization [21] where δ is suppressed by $1/m_b$ or by α_s , and is found to hold in a global SU(3) fit to $B \rightarrow PP$ [22]. This expected property and $P/T < 1$ imply [19] $\alpha < \alpha_{\text{eff}}$ and consequently

$$\alpha = [90_{-6}^{+7}(\text{exp}) \pm 6(\text{th})]^\circ. \quad (8)$$

2.3. Bounds on P/T from $B \rightarrow K^{*0}\rho^+$

It has been recently noted [8] that a stronger constraint on P/T in $B^0 \rightarrow \rho^+\rho^-$ may be obtained by relating this process to $B^+ \rightarrow K^{*0}\rho^+$ within flavor SU(3). The advantage of using this process over using $B^0 \rightarrow \rho^0\rho^0$ is twofold. First, penguin amplitudes in $\Delta S = 1$ decays are enhanced by a factor V_{cs}/V_{cd} relative to $\Delta S = 0$ decays. Second, $B^+ \rightarrow K^{*0}\rho^+$ is expected to be dominated by a penguin amplitude [23], whereas in $B^0 \rightarrow \rho^0\rho^0$ a penguin amplitude interferes with a potentially comparable or larger color-suppressed tree amplitude.

One uses both the branching ratio measured for this process [3], $\mathcal{B}(K^{*0}\rho^+) = (9.3 \pm 1.7) \times 10^{-6}$, and the measured fraction of longitudinal rate, $f_L(K^{*0}\rho^+) = 0.48_{-0.08}^{+0.09}$, to define a CP-averaged longitudinal amplitude $A_L(K^{*0}\rho^+)$. Since this

strangeness changing process is dominated by a penguin amplitude which is related by SU(3) to P in $B^0 \rightarrow \rho^+ \rho^-$, a parameter F can be introduced defined by

$$|A_L(K^{*0} \rho^+)|^2 = F \left(\frac{|V_{cs}| f_{K^*}}{|V_{cd}| f_\rho} \right)^2 P^2. \quad (9)$$

For a given value of F , supplementing $\Gamma_L(\rho^+ \rho^-)$, $S_L(\rho^+ \rho^-)$, $C_L(\rho^+ \rho^-)$ by $\Gamma_L(K^{*0} \rho^+)$, permits a determination of α , up to a discrete ambiguity [8]. The uncertainty in F is the source for a theoretical error in α .

The parameter F equals one in the limit of a purely factorized penguin amplitude. The ratios $|V_{cs}|/|V_{cd}|$ and f_{K^*}/f_ρ describe the corresponding CKM factors and an SU(3) breaking factor. Corrections to $F = 1$ are expected to be small. They follow from non-factorized SU(3) breaking (or form factor effects) and from two small terms [23], a color-suppressed electroweak penguin amplitude (P_{EW}^c) and a penguin annihilation contribution (PA) which is formally $1/m_b$ -suppressed. [Such a contribution would show-up in longitudinally polarized $B_s \rightarrow \rho^+ \rho^-$ decays [23].] A random scan through the input parameter space describing a model for these contributions in a QCD-factorization calculation [24] permits a rather broad range, $0.3 \leq F \leq 1.5$, favoring values smaller than one over values larger than one. We shall allow an even broader range which is symmetric around $F = 1$,

$$0.3 \leq F \leq 3.0. \quad (10)$$

This range, which we consider very conservative, will be used to demonstrate the low sensitivity of the error in α to the uncertainty in F .

To appreciate the advantage of this method over the isospin method, we note that the measurement of $\Gamma_L(K^{*0} \rho^+)$ implies a smaller value for P/T than implied by $\mathcal{B}(\rho^0 \rho^0)$. A value $F = 1$ corresponds to $P/T = 0.09$, which is considerably smaller than the upper bound $P/T < 0.3$ obtained from $\mathcal{B}(\rho^0 \rho^0)$. Smaller values of F imply a somewhat larger P/T . However, a value $P/T = 0.3$ would require $F = (0.09/0.3)^2 = 0.09$ which is unreasonably small. The main point here is therefore the following. *Once P has been established to*

be small, a large relative uncertainty in this amplitude, obtained by assuming flavor SU(3) and neglecting smaller terms, leads to only a small uncertainty in α . This expectation is demonstrated by fitting $\Gamma_L(\rho^+ \rho^-)$, $S_L(\rho^+ \rho^-)$, $C_L(\rho^+ \rho^-)$ and $\Gamma_L(K^{*0} \rho^+)$ in terms of T, P, δ and α while varying F in the range (10). The final result is

$$\alpha = [91_{-7}^{+9}(\text{exp})_{-4}^{+2}(\text{th})]^\circ, \quad (11)$$

where the theoretical error corresponds to the range (10). A discrete ambiguity between two values of α corresponding to $|\delta| \leq \pi/2$ and $|\delta| > \pi/2$ has been resolved by assuming $|\delta| \leq \pi/2$, as has already been assumed when applying the isospin method.

2.4. Averaged value of α from $B^0 \rightarrow \rho^+ \rho^-$

The two ways described in the previous subsections for extracting α in $B^0(t) \rightarrow \rho^+ \rho^-$ provide two independent constraints on the effect of the penguin amplitude on the asymmetries $S_L(\rho^+ \rho^-)$ and $C_L(\rho^+ \rho^-)$. Taking the average of (8) and (11) one has

$$\alpha = [91 \pm 7(\text{exp}) \pm 3(\text{th})]^\circ. \quad (12)$$

This value assumes that δ lies in the positive semicircle, $|\delta| < \pi/2$, an assumption motivated by QCD factorization and by a global SU(3) fit to $B \rightarrow PP$. This weak assumption is expected to be relaxed further by improving the precision of $C_L(\rho^+ \rho^-)$, which would eventually require excluding only values of δ near π . The assumption $|\delta| < \pi/2$ can be tested by fitting within SU(3) decay rates and CP asymmetries for all $B^{+,0} \rightarrow \rho \rho, K^* \rho$ decays for longitudinally polarized vector mesons.

The value (12) is consistent with values obtained in two global fits to all other CKM constraints [20,25] including the recent measurement of Δm_s [26]. Our extracted value is more precise than a value, $\alpha = (100_{-9}^{+15})^\circ$ [20], obtained in a fit combining $B \rightarrow \rho \rho, \pi \pi, \rho \pi$. The result (12) lies on the low side of this range because it assumes $|\delta| < \pi/2$. Including $B \rightarrow \pi \pi, \rho \pi$ would affect the average (12) only slightly since the error in (7) and errors of similar magnitudes involved in studies of $B \rightarrow \rho \pi$ [27,28] are considerably larger than the errors in (8) and (11).

The rather small theoretical error in α , $\pm 3^\circ$, implies that in studies using isospin symmetry one must consider isospin breaking effects which are expected to be of similar magnitude. As mentioned, in $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ the effect of electroweak penguin amplitudes on the isospin analyses has been calculated and was found to be $\Delta\alpha_{\text{EWP}} = -1.5^\circ$. Other effects include π^0 - η - η' mixing which affects the isospin analysis of $B \rightarrow \pi\pi$ [29] (a correction smaller than 1° was calculated in [30]), ρ - ω mixing affecting the $\pi^+\pi^-$ invariant-mass distribution in $B^+ \rightarrow \rho^+\rho^0$ [30], and a correction to the isospin analysis of $B \rightarrow \rho\rho$ from a potential $I = 1$ final state when the two ρ mesons are observed with different invariant-masses [31]. We note that the extraction of α by applying flavor SU(3) to $B^0 \rightarrow \rho^+\rho^-$ and $B^+ \rightarrow K^{*0}\rho^+$ involves only charged ρ mesons, and is therefore not susceptible to correction of this kind.

3. PRECISION $B \rightarrow K\pi$ SUM RULES

The decays $B \rightarrow K\pi$, which are dominated by a $\bar{b} \rightarrow \bar{s}q\bar{q}$ penguin amplitude, are sensitive to physics beyond the Standard Model because new heavy particles may replace the W boson and the top quark in the loop. In the Standard Model isospin symmetry implies relations among amplitudes, among rates and among CP asymmetries in $B \rightarrow K\pi$ decays. Relations of this kind are very useful as their violation would provide evidence for new physics. Sum rule for rates test the flavor structure of the effective weak Hamiltonian. Symmetry relations among CP asymmetries are particularly interesting because almost any extension of the model involves new sources of CP violation which lead to potential deviations from the sum rules.

3.1. Isospin in $B \rightarrow K\pi$

The four physical $B \rightarrow K\pi$ decay amplitudes are expressed in terms of three isospin-invariant amplitudes [32],

$$\begin{aligned} -A(K^+\pi^-) &= B_{1/2} - A_{1/2} - A_{3/2}, \\ A(K^0\pi^+) &= B_{1/2} + A_{1/2} - A_{3/2}, \\ -\sqrt{2}A(K^+\pi^0) &= B_{1/2} + A_{1/2} - 2A_{3/2}, \end{aligned}$$

$$\sqrt{2}A(K^0\pi^0) = B_{1/2} - A_{1/2} + 2A_{3/2}, \quad (13)$$

where B, A correspond to $\Delta I = 0, 1$ parts of \mathcal{H}_{eff} , respectively, while subscripts denote the isospin of the final $K\pi$ state. This implies a quadrangle relation,

$$\begin{aligned} \Sigma A(K\pi) &\equiv A(K^+\pi^-) - A(K^0\pi^+) \\ -\sqrt{2}A(K^+\pi^0) + \sqrt{2}A(K^0\pi^0) &= 0. \end{aligned} \quad (14)$$

This relation holds separately for B and \bar{B} decays for any linear combination of two arbitrary $\Delta I = 0$ and $\Delta I = 1$ transition operators. This feature turns out to be crucial when discussing isospin breaking effects in $B \rightarrow K\pi$ sum rules.

3.2. Precise sum rule for $B \rightarrow K\pi$ rates

The following relation is obeyed approximately for $B \rightarrow K\pi$ decay rates [9,10],

$$\begin{aligned} \Gamma(K^+\pi^-) + \Gamma(K^0\pi^+) &\approx \\ 2\Gamma(K^+\pi^0) + 2\Gamma(K^0\pi^0). \end{aligned} \quad (15)$$

This sum rule holds up to terms which are quadratic in small quantities. The proof of this sum rule is rather simple. The dominant isospin amplitude is the singlet $B_{1/2}$, the only one containing a penguin amplitude. Terms which are linear or quadratic in $B_{1/2}$ involve the product of this amplitude with the sum $\Sigma A(K\pi)$ which vanishes. The remaining terms are quadratic in two small ratios of tree (T) or electroweak penguin amplitudes (P_{EW}) and the dominant penguin amplitude (P). The two ratios, T/P and P_{EW}/P , are between 0.1 and 0.2. Thus, small corrections to the sum rule were calculated and were found to be between one and five percent [24,33,34,35]. A larger deviation would require an anomalously large $\Delta I = 1$ operator in the effective Hamiltonian.

At the level of precision expected in the Standard Model one must consider also isospin breaking corrections to the sum rule. In general one would expect these corrections to be linear in isospin breaking, namely of order $(m_d - m_u)/\Lambda_{\text{QCD}} \simeq 0.03$. As it turns out, isospin breaking corrections in the sum rule are further suppressed by the small ratio T/P . The argument for this suppression holds for both Eqs. (14) and

(15) [13] and is explained briefly in the next two paragraphs.

The spurion representing isospin breaking caused by the mass and charge-difference of the d and u quarks behaves like a sum of $\Delta I = 0$ and $\Delta I = 1$ operators. Thus, in the presence of isospin breaking the dominant isosinglet amplitude $B_{1/2}$ becomes a sum of $\Delta I = 0$ and $\Delta I = 1$ amplitudes. Since Eq. (14) holds for an arbitrary combination of isosinglet and isotriplet operators, it holds for the dominant terms also when isospin breaking is included. Thus, isospin breaking appears only in subdominant terms in (14) and is suppressed by T/P .

Similarly, the dominant terms in Eq. (15) are quadratic in $B_{1/2}$ and their linear isospin breaking term is a combination of isosinglet and isotriplet contributions. The sum of contributions appearing in (15) vanishes as it involves the same combination as (14). Since isospin breaking corrections cancel in (15) in terms which are quadratic in $B_{1/2}$, the remaining isospin breaking is suppressed by T/P (or by $A_{1/2,3/2}/B_{1/2}$). This correction is significantly smaller than that of the quadratic terms correcting the sum rule which are between one and five percent.

The current experimental situation of the sum rule (15) can be summarized in terms of branching ratios corrected by the B^+/B^0 lifetime ratio, $\tau_+/\tau_0 = 1.076 \pm 0.008$ [3],

$$44.4 \pm 1.5 \approx 48.9 \pm 2.7, \quad (16)$$

which works within 1.5σ . Two kinds of isospin breaking effects in branching ratio measurements must be studied more carefully: (i) Radiative corrections [36] which have not been included in all $B \rightarrow K\pi$ measurements. (ii) The effect on branching ratio measurements of a small isospin-breaking difference between the production rates of B^+B^- and $B^0\bar{B}^0$ pairs at the $\Upsilon(4S)$ [37].

3.3. Success of SU(3) in CP asymmetries

CP asymmetries in $B \rightarrow K\pi$ decays have been the subject of a large number of theoretical studies, whose most difficult parts were model-dependent calculations of strong phases. While it is hard to calculate magnitudes of asymmetries, one may compare several asymmetries us-

ing symmetry arguments. Of the four $B \rightarrow K\pi$ asymmetries only one, that measured in $B^0 \rightarrow K^+\pi^-$ is significantly different from zero [3], $A_{\text{CP}}(K^+\pi^-) = -0.108 \pm 0.017$. It is interesting to compare this asymmetry with a second nonzero asymmetry measured in $B^0 \rightarrow \pi^+\pi^-$, $A_{\text{CP}}(\pi^+\pi^-) = 0.37 \pm 0.10$. (The error does not reflect a certain disagreement between Babar and Belle measurements.) In flavor SU(3), the two processes involve common tree and penguin amplitudes multiplied by different CKM factors. While the amplitude of $B^0 \rightarrow \pi^+\pi^-$ is given by (1), that of $B^0 \rightarrow K^+\pi^-$ is

$$A(B^0 \rightarrow K^+\pi^-) = \lambda T e^{i\gamma} - \lambda^{-1} P e^{i\delta}, \quad (17)$$

where $\lambda = V_{us}/V_{ud}$. Consequently, the two CP rate differences have equal magnitudes and opposite signs, and the asymmetries are related in a reciprocal manner to the corresponding branching ratios [23,38],

$$\begin{aligned} \frac{A_{\text{CP}}(\pi^+\pi^-)}{A_{\text{CP}}(K^+\pi^-)} &= -\frac{\mathcal{B}(K^+\pi^-)}{\mathcal{B}(\pi^+\pi^-)}, \\ -3.4 \pm 1.1 &= -4.0 \pm 0.4. \end{aligned} \quad (18)$$

The agreement of signs and magnitudes supports the assumption that weak hadronic amplitudes and strong phases are approximately SU(3) invariant. Deviations at a level of 30% are expected in (18) from SU(3) breaking corrections and from $1/m_b$ -suppressed annihilation amplitudes.

3.4. Precise sum rule for $K\pi$ asymmetries

Isospin symmetry can be applied to the four $K\pi$ asymmetries to obtain an approximate relation [11,12,39],

$$2(\Delta_{+0} + \Delta_{00}) \approx \Delta_{+-} + \Delta_{0+}, \quad (19)$$

where one defines CP rate differences $\Delta_{ij} \equiv \Gamma(B \rightarrow K^i\pi^j) - \Gamma(\bar{B} \rightarrow K^{\bar{i}}\pi^{\bar{j}})$. The proof of this sum rule is based on the fact that each CP rate difference can be written as a product of an imaginary part of products of hadronic amplitudes and an invariant imaginary part of products of CKM factors, $4\text{Im}(V_{tb}^*V_{ts}V_{ub}V_{us}^*)$. Writing the dominant term in Δ_{ij} symbolically as $\text{IM}[P^*A(K^i\pi^j)]$, where P dominates each of the $K\pi$ amplitudes, the dominant term in the difference between the

left-hand side and the right-hand-side of (19) is $\text{IM}[P^*\Sigma A(K\pi)]$ which vanishes by (14). The subdominant terms in this difference, involving terms of the form $\text{IM}(P_{EW}^*T)$, are suppressed by about an order of magnitude relative to $\text{IM}(P^*T)$ and can be shown to cancel in the flavor SU(3) limit.

In the penguin dominance approximation, $\Gamma(K^+\pi^-) \approx \Gamma(K^0\pi^+) \approx 2\Gamma(K^+\pi^0) \approx 2\Gamma(K^0\pi^0)$, the relation (19) simplifies to a sum rule among corresponding CP asymmetries,

$$\begin{aligned} A_{\text{CP}}(K^+\pi^0) + A_{\text{CP}}(K^0\pi^0) &\approx \\ A_{\text{CP}}(K^+\pi^-) + A_{\text{CP}}(K^0\pi^+). \end{aligned} \quad (20)$$

This sum rule is expected to hold to any foreseeable experimental precision, the most difficult asymmetry being that of $B^0 \rightarrow K^0\pi^0$. Using the currently measured asymmetries [3], it reads

$$\begin{aligned} (0.04 \pm 0.04) + (0.02 \pm 0.13) &\approx \\ (-0.108 \pm 0.017) + (-0.02 \pm 0.04), \end{aligned} \quad (21)$$

which holds within experimental errors. Using three of the measured asymmetries, the sum rule (19) predicts $A_{\text{CP}}(K^0\pi^0) = -0.15 \pm 0.06$.

Before closing this section we wish to comment briefly on a so-called ‘‘puzzle’’ which is sometimes being claimed by observing $A_{\text{CP}}(K^+\pi^0) \neq A_{\text{CP}}(K^+\pi^-)$. The approximation $A_{\text{CP}}(K^+\pi^0) \sim A_{\text{CP}}(K^+\pi^-)$ was suggested several years ago [9] based on classifying contributions to $B \rightarrow K\pi$ amplitudes in terms of flavor topologies [23], where a hierarchy $C \ll T$ was assumed between color-allowed and color-suppressed tree amplitudes. In fact, there exists no compelling theoretical argument for a suppression of C relative to T . A global SU(3) analysis of $B \rightarrow PP$ [22,40] indicates that the two contributions are comparable. An example for a sizable C [41] is the large $\mathcal{B}(\pi^0\pi^0)$. Abandoning the assumption $C \ll T$, the two asymmetries $A_{\text{CP}}(K^+\pi^0)$ and $A_{\text{CP}}(K^+\pi^-)$ could be different in the Standard Model. The sum rule (19) (or a similar sum rule in which a small Δ_{0+} is neglected [11]) holds also for a sizable C .

4. CONCLUSION

The currently most precise extraction of α based on $B^0 \rightarrow \rho^+\rho^-$ involves a theoretical er-

ror of a few degrees ($\pm 3^\circ$), requiring the inclusion of isospin breaking corrections where applicable. Two sum rules were studied based on isospin symmetry, involving $B \rightarrow K\pi$ decay rates and CP asymmetries in these processes. Isospin breaking in the first sum rule was shown to be suppressed and is therefore negligible, while the second sum rule is expected to hold within any foreseeable experimental accuracy.

Both sum rules are unaffected by a new isoscalar operator, which could simply be added to the dominant penguin amplitude. A potential violation of the sum rules would therefore be evidence for an anomalous $\Delta I = 1$ operator in the effective Hamiltonian. Observing such a violation requires reducing experimental errors in $B \rightarrow K\pi$ rates and asymmetries by at least a factor two.

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